

Validation of a MHD Module for Conductive Fluids in Square Duct up to High Hartmann Numbers

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Résumé :

Un module MHD est présenté, développé par la société Fluidyn, dans la plateforme multiphysique Fluidyn-MP. Ce module est basé sur une résolution des équations de Navier-Stokes incluant les forces de Lorentz qui apparaissent lorsqu'un fluide conducteur est soumis à l'action d'un champ magnétique externe continu. Le module est développé de telle sorte qu'il puisse simuler de manière précise des écoulements soumis à des champs magnétiques élevés, c'est-à-dire pour des nombre de Ha de l'ordre de $10^4 - 10^5$. Deux formulations sont proposées : formulation en potentiel électrique, et formulation en champ magnétique induit, validés premièrement pour des cas de calculs académiques d'écoulements en conduites, et comparés à des solutions analytiques de Hunt. Les deux méthodes donnent de très bons résultats jusqu'à $Ha=30000$.

Abstract :

A CFD-based MHD module is presented, which has been implemented into Fluidyn-MP multiphysics platform. This specific module is based on a resolution of the incompressible Navier-Stokes equations for conducting fluids submitted to external constant (DC) magnetic fields. Fluid flow is solved up to high magnetic fields, with two different numerical solution approaches: (1) potential and (2) induced magnetic field formulations. Both methods are implemented and tested on Hunt's benchmarks and found to be giving very good results in terms of velocity profiles

Mots clefs : MHD, Duct flows, Numerical simulation, inductive method, CFD, Fluid-structure interaction, potential method

1 Introduction

1.1 Foreword – Frame of present exercise

Fluidyn has been engaging in the past 3 years, a key-development in the field of CFD and fluid-structure coupling, implementing a MHD module into Fluidyn-MP platform. This module emphasizes accurate solution of velocity profile and current distribution for MHD fluid flows submitted to high magnetic fields and in 3D complex geometries of industrial configurations. MHD module comprises 2 different methods which are presented in the present paper. Fluid solvers and coupling are optimized, such as to reach sufficient accuracy for both methods. Industrial applications are performed with this module essentially for nuclear applications, such as ITER blanket cooling processes under strong magnetic field appliance.

1.2 Case description

In the present paper, both inductive and potential methods are compared on Hunt's academic rectangular duct flow solutions [1], with varying wall conductivities. Geometry of the domain is depicted in figure 1.

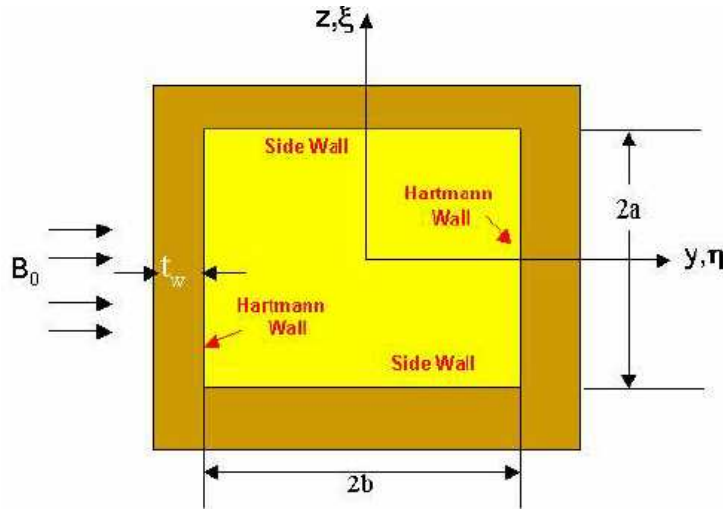


FIG. 1 – Hunt's duct flow cross section configuration (wall nomenclature and coordinates)

An electrically conducting fluid flows inside a square duct and exposed to a constant magnetic field. The flow is assumed to be incompressible, isothermal, and fully developed (laminar flow). The electrical boundary conditions on the walls of the duct are varied resulting in different scenarios. Both the electric potential method and the magnetic induction method are used with an objective of identifying the suitable method required for further analysis.

In figure 1 of Hunt's problem, geometry and process parameters are defined as follows:

$$2a = 0.2\text{m}, 2b = 0.2\text{m}, t_w = 0.001\text{m}, B_0 = \text{magnetic field (T)}$$

Magnetic field varies with Hartmann Number (Ha), as:

$$Ha = \left(\frac{\text{Electromagnetic forces}}{\text{Viscous forces}} \right)^{1/2} = B_0 L \sqrt{\frac{\sigma}{\mu}}$$

Where, L = Length = 0.1m
 σ = electric conductivity = 7.5e5 S/m
 μ = viscosity = 0.002116 Pa.s

Sl. No.	Ha	B_0 T
1	15000	7.967
2	25000	13.279
3	30000	15.935

A parametric study is performed in present benchmarking exercise, where the varying parameters are: B_0 (Ha), and conductivities of the different walls (side walls and Hartmann walls). Also, both potential and induced magnetic field method are used. Section 2 describes shortly the MHD module and both methods used, while section 3 reports details on results in terms of velocity profiles in the cross section of duct.

2 MHD module description

Magnetohydrodynamics (MHD) refers to the interaction between an applied electromagnetic field and a flowing, electrically-conductive fluid. The coupling between the fluid flow field and the magnetic field can be understood on the basis of two fundamental effects: the induction of electric current due to the movement of conducting material in a magnetic field, and the effect of Lorentz force as the result of electric current and magnetic field interaction. In general, the induced electric current and the Lorentz force tend to oppose the mechanisms that create them. The Navier Stokes

equation for the flow of an electrically conducting fluid subject to a magnetic field is written as:

$$\frac{\partial(\rho \mathbf{U})}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) = \nabla \cdot \bar{\bar{\tau}} - \nabla p + \rho \mathbf{g} + \mathbf{J} \times \mathbf{B}$$

where $\bar{\bar{\tau}} = \mu \dot{\gamma} - (\frac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{U})\delta$, the viscous stress tensor
 $\dot{\gamma} = \nabla \mathbf{U} + (\nabla \mathbf{U})^T$, the rate-of-strain (or rate-of-deformation) tensor
 μ = effective viscosity
 κ = dilatational viscosity (=0 according to Stokes)
 δ = unit tensor
 \mathbf{J} = current density vector
 \mathbf{B} = total magnetic field (induced + external)

Generally, two approaches are used to evaluate the current density, j :

1. One is through the solution of a magnetic induction equation; and
2. The other is through solving an electric potential equation.

2.1 Magnetic Induction Method

The magnetic induction equation that is derived from the Maxwell's equations:

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{U} - \nabla \times \left[\frac{1}{\sigma} \nabla \times \left(\frac{\mathbf{B}}{\mu_m} \right) \right]$$

where μ_m = magnetic permeability
 σ = electric conductivity

which can be expressed, using vector identities, as a convective-diffusive equation generally used in CFD:

$$\begin{aligned} \frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{U} \mathbf{B}) &= \frac{1}{\mu_m \sigma} \nabla \cdot (\nabla \mathbf{B}) + (\mathbf{B} \cdot \nabla) \mathbf{U} + \mathbf{U} (\nabla \cdot \mathbf{B}) \\ &+ \frac{1}{\mu_m \sigma} \left\{ \nabla (\mu_m \sigma) \times (\mathbf{E} + \mathbf{U} \times \mathbf{B}) + \nabla \times \left(\frac{1}{\mu_m} \nabla \mu_m \times \mathbf{B} \right) \right\} \end{aligned}$$

The terms in flower brackets are normally ignored. Current density, \mathbf{J} , is computed using Ampère's law:

$$\mathbf{J} = \nabla \times \left(\frac{\mathbf{B}}{\mu_m} \right)$$

2.2 Potential Method

The current density, \mathbf{J} , is expressed through the generalized Ohm's law and assuming negligible induced magnetic field, as:

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{U} \times \mathbf{B}_0)$$

where φ = electric potential
 \mathbf{B}_0 = external magnetic field

$$\begin{aligned}\mathbf{E} &= \text{electric field} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} \quad \text{Helmholtz's theorem.} \\ \mathbf{A} &= \text{vector field}\end{aligned}$$

For a static field, the Ohm's law can be simplified as:

$$\mathbf{J} = \sigma(-\nabla\varphi + \mathbf{U} \times \mathbf{B}_0)$$

The conservation of electric charges gives,

$$\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

The electric potential equation is then given by,

$$\nabla \cdot [\sigma(\nabla\varphi)] = \nabla \cdot [\sigma(\mathbf{U} \times \mathbf{B}_0)]$$

3 Implementation of MHD methods in Fluidyn

3.1 Magnetic Induction Method

The magnetic induction method has been implemented in the NSNT module with the following boundary conditions:

Boundary condition	For an insulating boundary,	For a perfectly conducting boundary,	For a partially conducting boundary,
Imposed boundary condition	$\mathbf{B} = \mathbf{B}_0$	$\frac{\partial \mathbf{B}}{\partial n} = 0$	$\frac{\partial \mathbf{B}}{\partial n} = \frac{\mathbf{B}}{c_w}, c_w = \frac{\sigma_w t_w}{\sigma a}$

Where \mathbf{B}_0 represents external magnetic field.

3.2 Potential Method

The electric potential method has been implemented in the NSNT module with the following boundary conditions:

Boundary condition	For an insulating boundary,	For a perfectly conducting boundary,
Imposed boundary condition	$\frac{\partial \varphi}{\partial n} = (\vec{V} \times \vec{B}_0)_b \cdot \vec{n}$	$\varphi = \varphi_o$

Where φ_o is the specified potential at the boundary.

4 Parametric results

4.1 Input data

Following test cases have been performed on previously described Hunt's duct flow configuration, on a structured mesh, on several configurations of insulation or conductive side walls and Hartmann walls. For simplicity, only some of the results are described hereafter for both inductive and potential method. described in figure 2:

MHD Method	Sl. No.	Task	Ha	Hartmann Wall	Side Wall
Induction	1	1.1A.11a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
	2	1.1A.11b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
	3	1.1A.11c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
	4	1.1A.12a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$
	5	1.1A.12b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$
	6	1.1A.12c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$
	7	1.1A.13a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.1\sigma_b/t_w$
	8	1.1A.13b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.1\sigma_b/t_w$
	9	1.1A.13c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.1\sigma_b/t_w$
MHD Method	Sl. No.	Task	Ha	Hartmann Wall	Side Wall
Induction	1	1.1B.11a	15000	$\sigma_{hw}=0$	$\sigma_{sw}=0$
	2	1.1B.11b	25000	$\sigma_{hw}=0$	$\sigma_{sw}=0$
	3	1.1B.11c	30000	$\sigma_{hw}=0$	$\sigma_{sw}=0$
	4	1.1B.12a	15000	$\sigma_{hw}=0.01\sigma_b/t_w$	$\sigma_{sw}=0$
	5	1.1B.12b	25000	$\sigma_{hw}=0.01\sigma_b/t_w$	$\sigma_{sw}=0$
	6	1.1B.12c	30000	$\sigma_{hw}=0.01\sigma_b/t_w$	$\sigma_{sw}=0$
	7	1.1B.13a	15000	$\sigma_{hw}=0.1\sigma_b/t_w$	$\sigma_{sw}=0$
	8	1.1B.13b	25000	$\sigma_{hw}=0.1\sigma_b/t_w$	$\sigma_{sw}=0$
	9	1.1B.13c	30000	$\sigma_{hw}=0.1\sigma_b/t_w$	$\sigma_{sw}=0$

MHD Method	Sl. No.	Task	Ha	Hartmann Wall	Side Wall
Potential	1	1.2A.11a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
	2	1.2A.11b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
	3	1.2A.11c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
	4	1.2A.12a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$
	5	1.2A.12b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$
	6	1.2A.12c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$
	7	1.2A.13a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.1\sigma_b/t_w$
	8	1.2A.13b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.1\sigma_b/t_w$
	9	1.2A.13c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.1\sigma_b/t_w$
MHD Method	Sl. No.	Task	Ha	Hartmann Wall	Side Wall
Potential	1	1.2B.11a	15000	$\sigma_{hw}=0$	$\sigma_{sw}=0$
	2	1.2B.11b	25000	$\sigma_{hw}=0$	$\sigma_{sw}=0$
	3	1.2B.11c	30000	$\sigma_{hw}=0$	$\sigma_{sw}=0$
	4	1.2B.12a	15000	$\sigma_{hw}=0.01\sigma_b/t_w$	$\sigma_{sw}=0$
	5	1.2B.12b	25000	$\sigma_{hw}=0.01\sigma_b/t_w$	$\sigma_{sw}=0$
	6	1.2B.12c	30000	$\sigma_{hw}=0.01\sigma_b/t_w$	$\sigma_{sw}=0$
	7	1.2B.13a	15000	$\sigma_{hw}=0.1\sigma_b/t_w$	$\sigma_{sw}=0$
	8	1.2B.13b	25000	$\sigma_{hw}=0.1\sigma_b/t_w$	$\sigma_{sw}=0$
	9	1.2B.13c	30000	$\sigma_{hw}=0.1\sigma_b/t_w$	$\sigma_{sw}=0$

FIG. 2 – Parametric configurations

Properties of materials both for fluid and structure are taken as in following table:

Property	Solid (Ferritic Martensitic Steel)	Fluid (Lead - Lithium)
Density (kg/m ³)	7871	8563.43
Electrical Conductivity (S/m)	Varies depending on the case	7.5e5
Viscosity (Pa.s)	--	0.002116

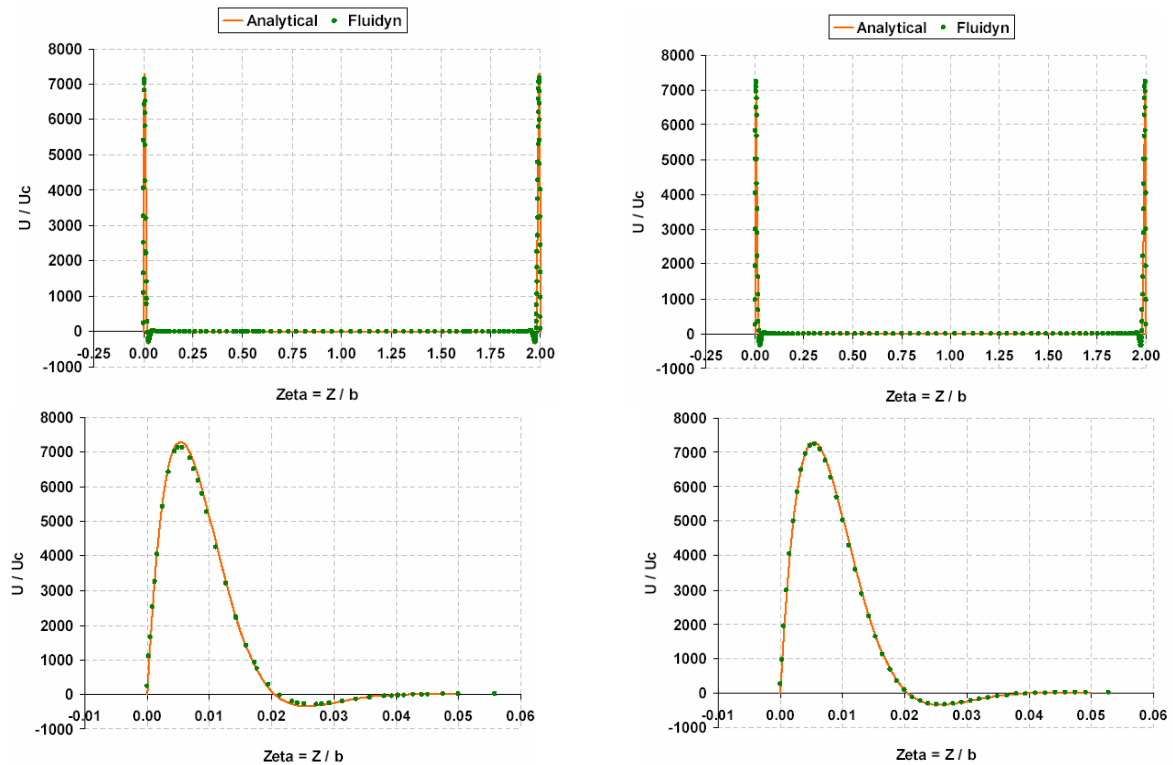
4.2 Parametric results

Case 1.1A.11c and 1.2A.11c are presented hereafter with following conditions:

1.1A.11c	Induction	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$
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Pressure

Gradient used for a mean velocity of 0.18m/s = 649400 Pa/m.



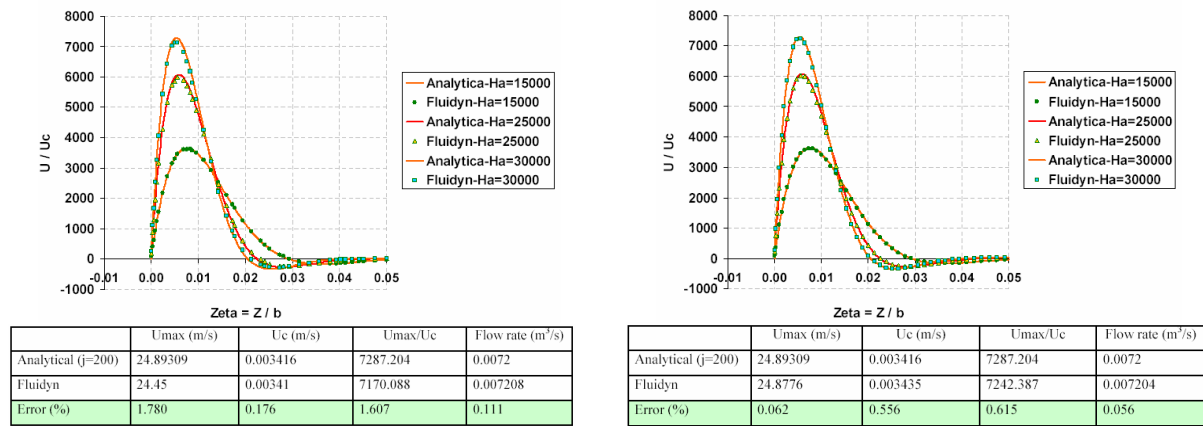


FIG. 3 – Velocity profile across side layer (full and zoomed view) + comparison of profiles for different Ha values. Left – induction model, Right – potential model

Summary:

Sl. No.	Task	Ha	Hartmann Wall	Side Wall	Analytical Uc m/s	Fluidyn Uc m/s	Error %
1	1.1A.11a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$	0.00480	0.00478	0.319
2	1.1A.11b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$	0.00375	0.00373	0.455
3	1.1A.11c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$	0.00343	0.00341	0.531
4	1.1A.12a	15000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$	0.00979	0.00978	0.085
5	1.1A.12b	25000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$	0.00876	0.00881	0.616
6	1.1A.12c	30000	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$	0.00846	0.00851	0.616

FIG. 4 – Summary of parametric study

5 Conclusions

Following conclusions and discussions were drawn after present benchmarking exercise:

- In general, the Fluidyn results match very well the analytical solutions: error < 1%.
- Both induction and potential methods yield results of similar accuracy.
- Most difficult case to converge numerically is the all-insulated case

Following tables show the comparison of no of cycles and CPU hours used in induction and potential method.

Sl. No.	Hartmann Wall	Side Wall	Ha	Induction Method		Potential Method	
				No of cycles	~CPU Time Hours	No of cycles	~CPU Time Hours
1	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$	15000	94000	32	83000	46
2	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$	25000	207000	58	265000	108
3	$\sigma_{hw}=\infty$	$\sigma_{sw}=0$	30000	1556000	288	276000	122
4	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$	15000	221000	54	280000	124
5	$\sigma_{hw}=\infty$	$\sigma_{sw}=0.01\sigma_b/t_w$	25000	227000	60	323000	154

References

- [1] Magnetohydrodynamic flow in rectangular ducts, **J. C. R. Hunt**, JFM, 1965
- [2] Fluidyn MP, NSNT solver, Users-manual, v5.2, July 2009.